# Spectral processing

# 10.1 Introduction

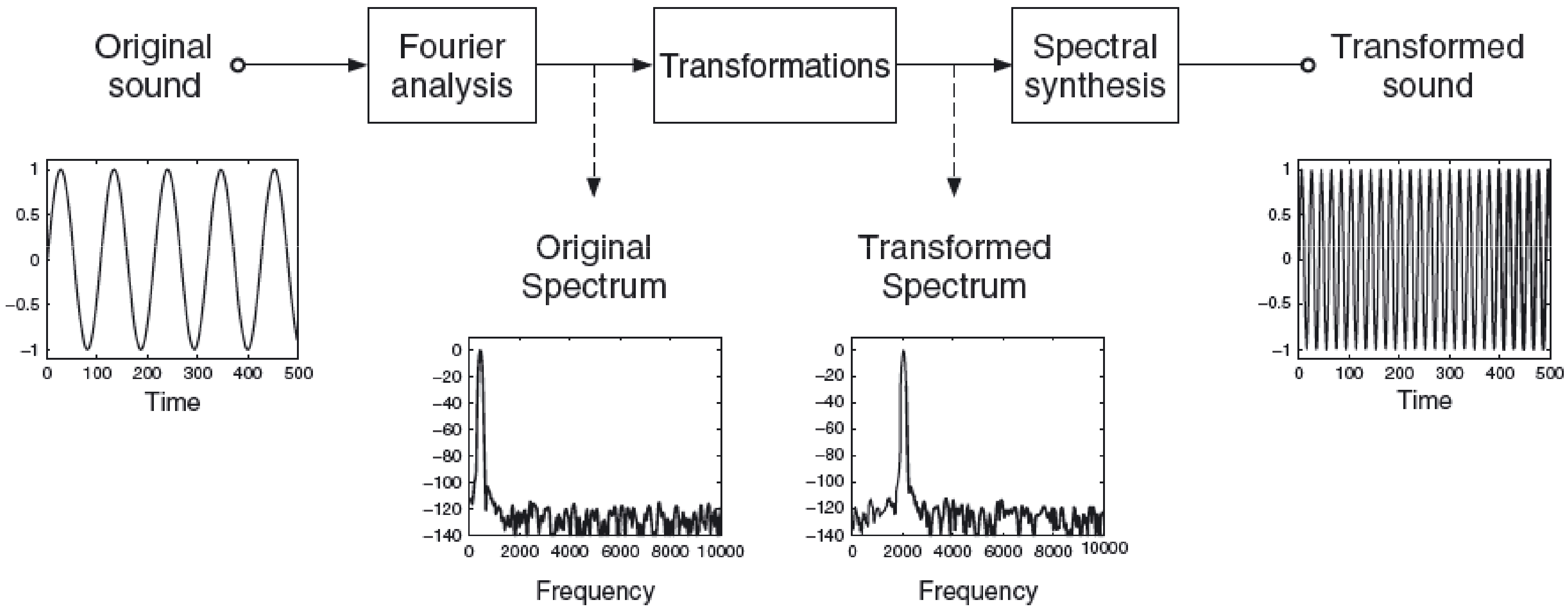


**Figure 10.1** Block diagram of a simple spectral-processing framework.

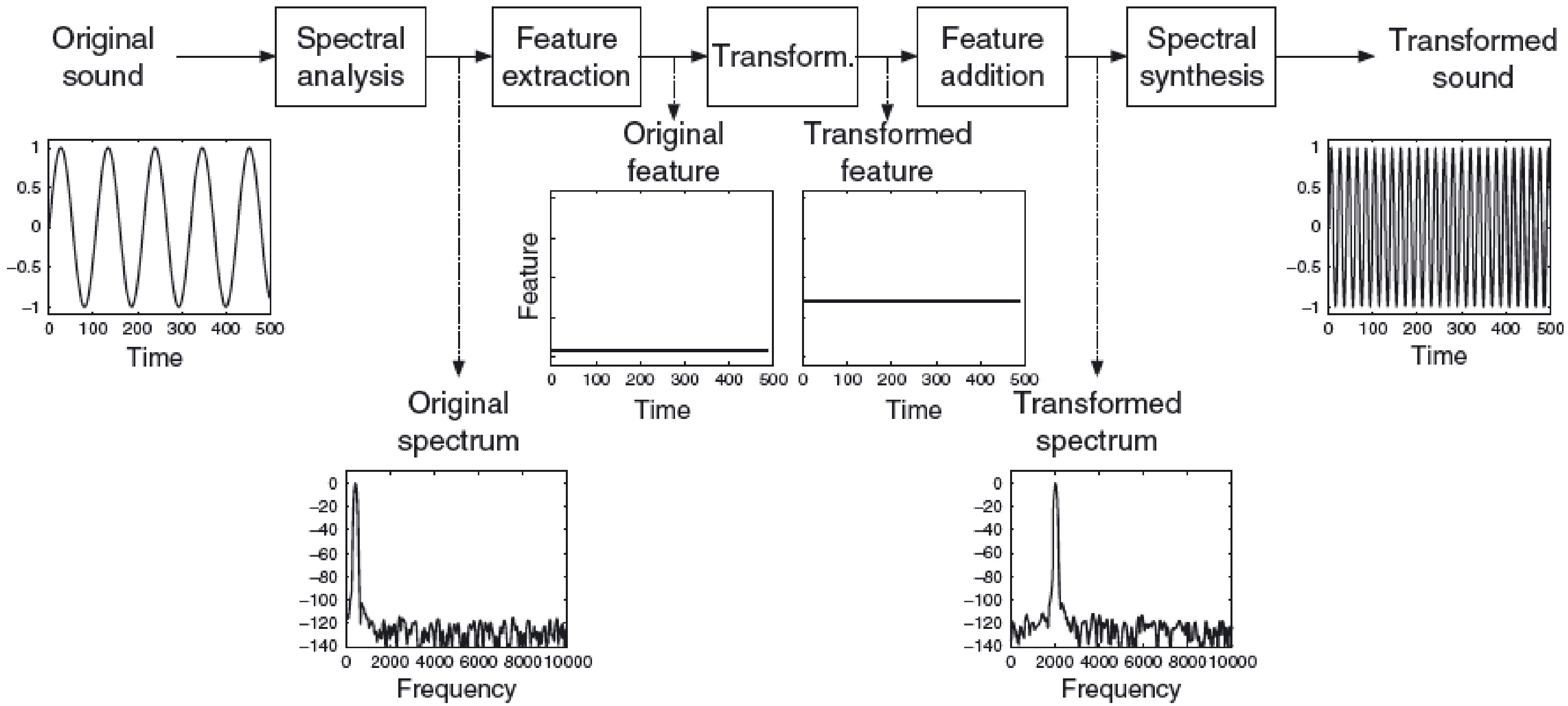


**Figure 10.2** Block diagram of a higher-level spectral-processing framework.

# 10.2 Spectral models

$$X_l(k) = \sum_{n=0}^{N-1} w(n) x(n + lH) \, \mathrm{e}^{-j\omega_k n}$$

# 10.2.1 Sinusoidal model

$$s(t) = \sum_{r=1}^{R} A_r(t) \cos[\theta_r(t)]$$

# 10.2.2 Sinusoidal plus residual model

$$s(t) = \sum_{r=1}^{R} A_r(t) \cos\left[\theta_r(t)\right] + e(t)$$

$$\theta_r(t) = \int_0^t \omega_r(\tau)\,\mathrm{d}\tau$$

$$e(t) = \int_0^t h(t, \tau)\, u(\tau)\,\mathrm{d}\tau$$

# 10.3 Techniques

## 10.3.1 Short-time fourier transform

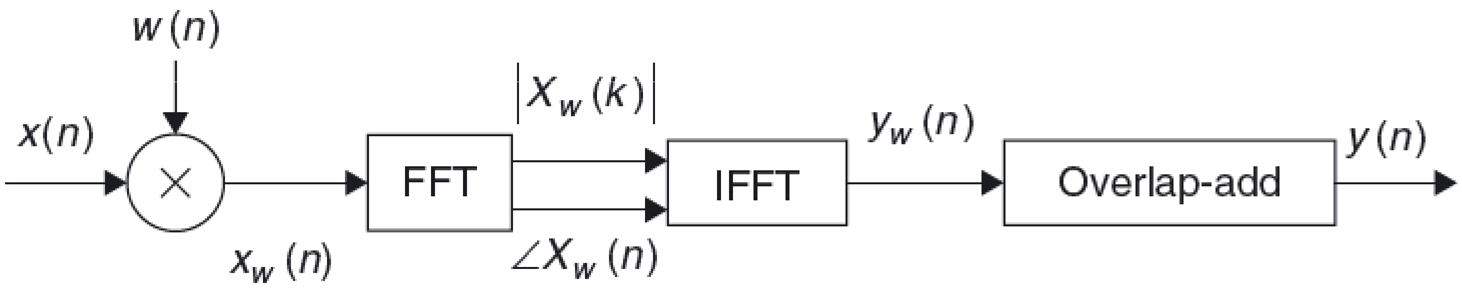


**Figure 10.3** Block diagram of an analysis/synthesis system based on the STFT.

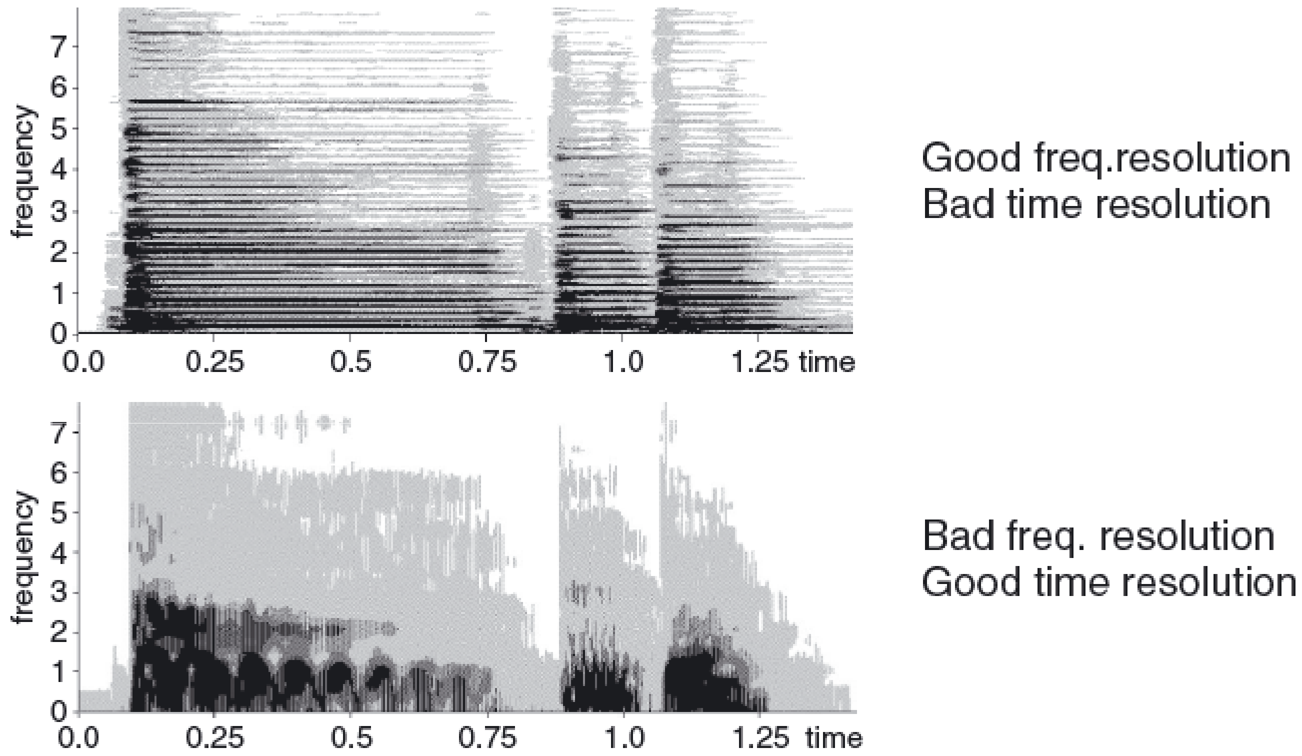


**Figure 10.4** Time vs. frequency resolution trade-off.

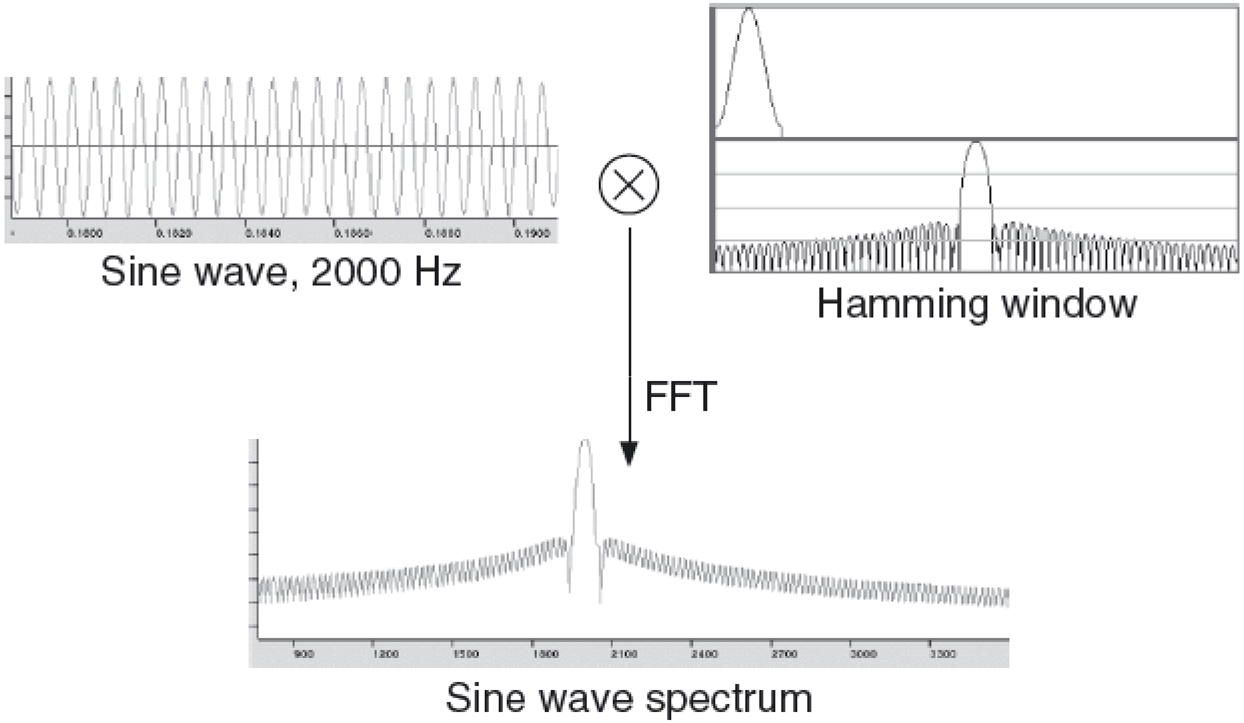


**Figure 10.5** Effect of applying a window in the time domain.

$$M \geq B_s \frac{f_s}{|f_{k+1} - f_k|}$$

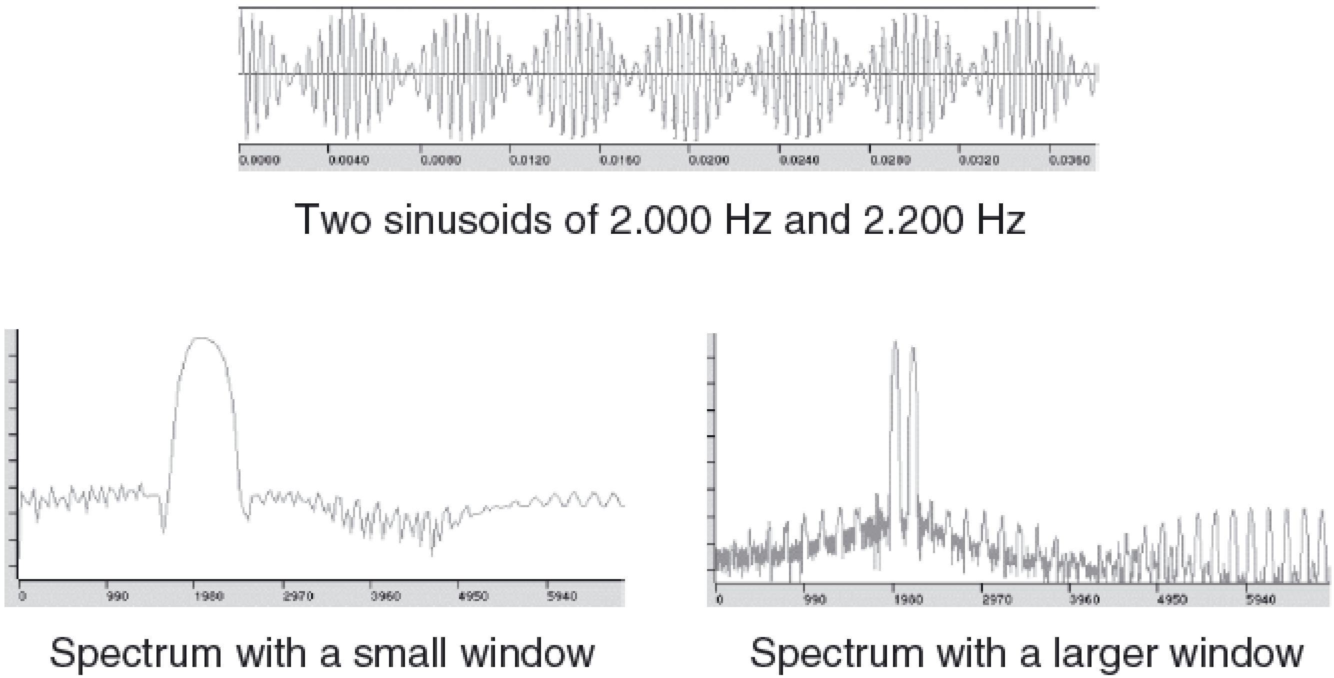


**Figure 10.6** Effect of the window size in distinguishing between two sinusoids.

$$A_w(m) = \sum_{n=-\infty}^{\infty} w(m - nH) \approx \text{constant}$$

$$d_w = 100 \times \frac{\max_w [A_w(m)] - \min_w [A_w(m)]}{\max_w [A_w(m)]}$$

# 10.3.2 Spectral peaks

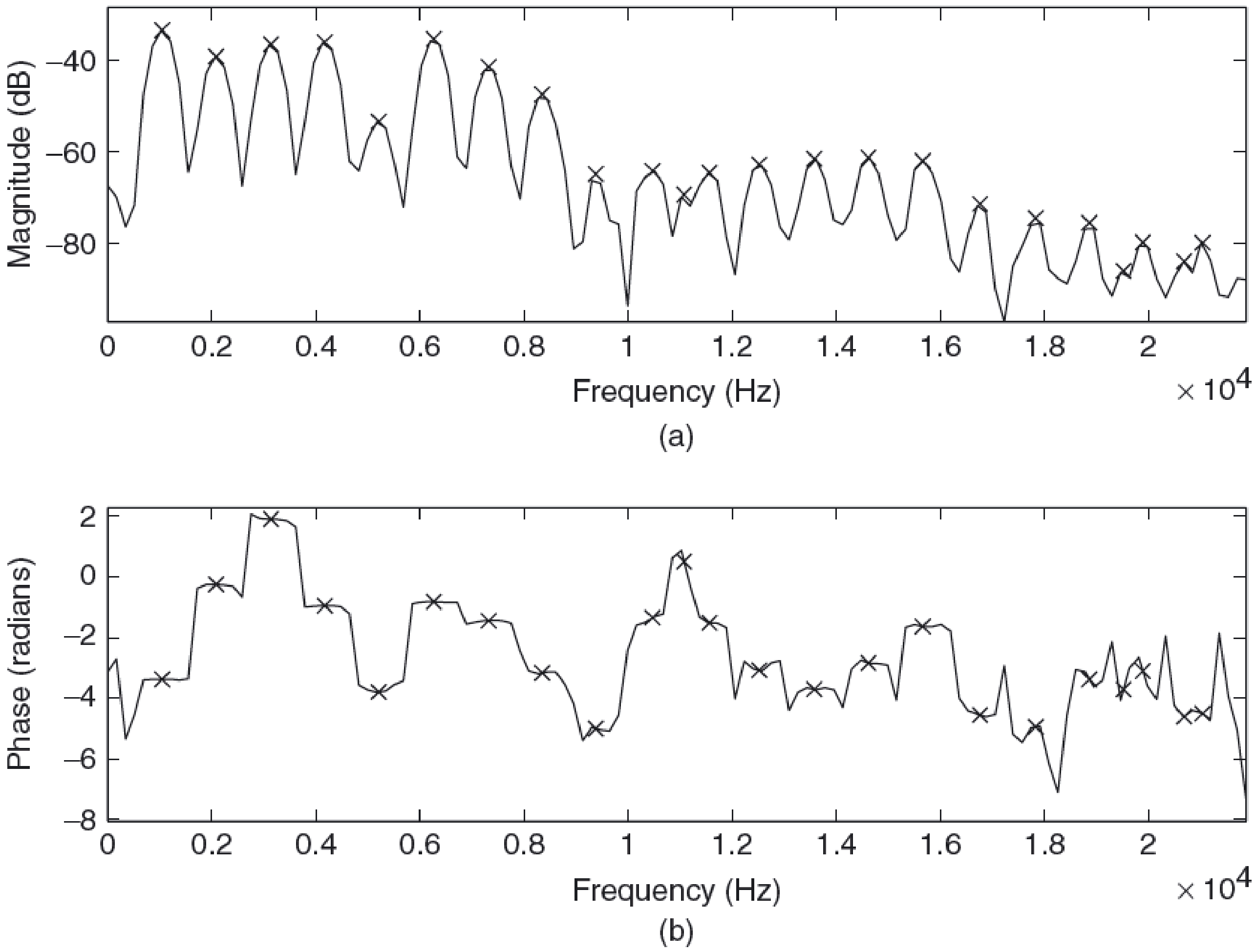


**Figure 10.7** Peak detection: (a) peaks in magnitude spectrum; (b) peaks in the phase spectrum.

# 10.3.3 Spectral sinusoids

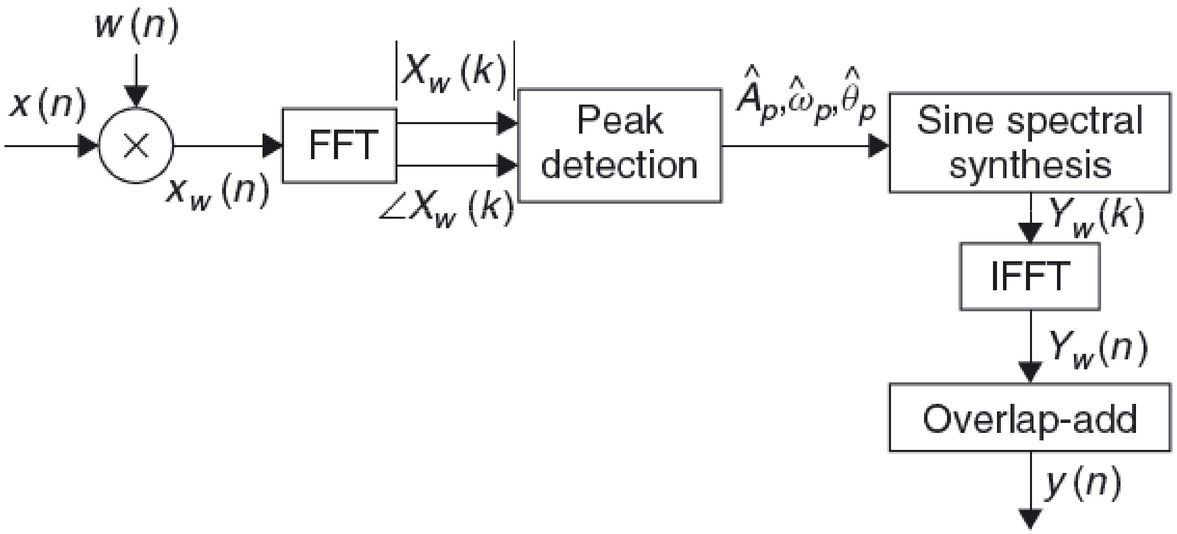


**Figure 10.8** Block diagram of an analysis/synthesis system based on the sinusoidal model.

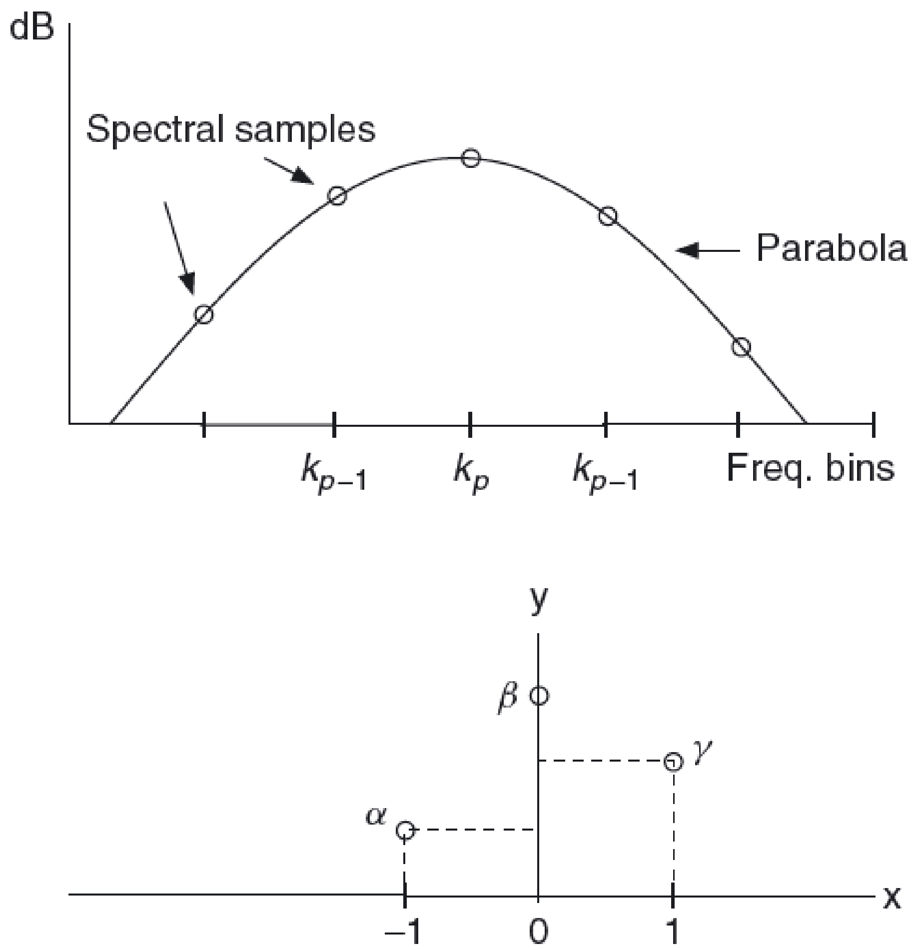


**Figure 10.9** Parabolic interpolation in the peak-detection process.

• **Peak continuation**

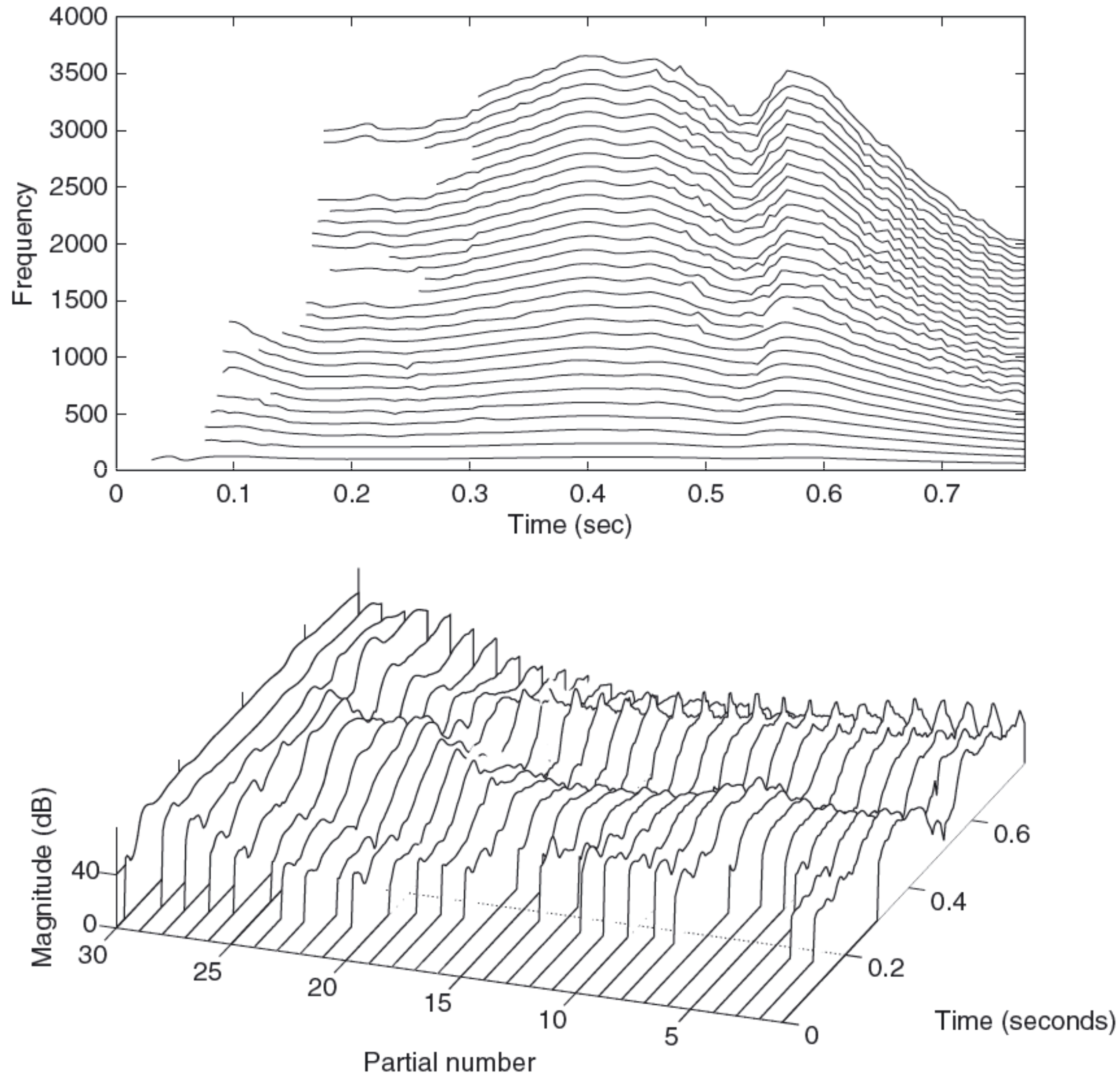


**Figure 10.10** Frequency trajectories resulting from the sinusoidal analysis of a vocal sound.

- **Sinusoidal synthesis**

$$\hat{A}^{l}(m) = \hat{A}^{l-1} + \frac{\left(\hat{A}^{l} - \hat{A}^{l-1}\right)}{H} m$$

$$\hat{\omega}^{l}(m) = \hat{\omega}^{l-1} + \frac{\left(\hat{\omega}^{l} - \hat{\omega}^{l-1}\right)}{H} m$$

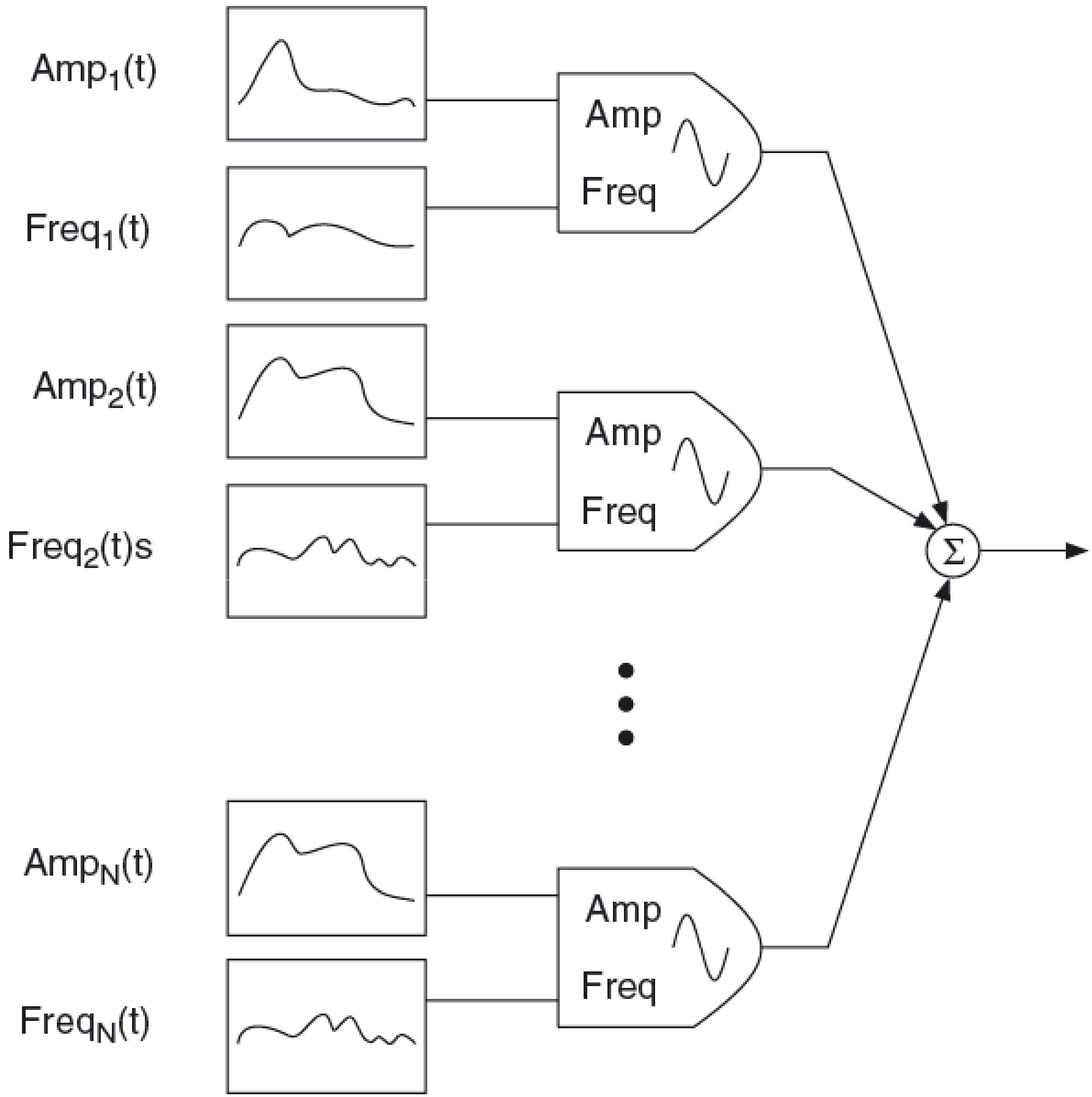


**Figure 10.11** Additive synthesis block diagram.

$$\hat{\theta}^l(m) = \hat{\theta}^{l-1}(H-1) + \sum_{s=0}^{m} \hat{\omega}^l(s)$$

$$d^l(m) = \sum_{r=1}^{R^l} \hat{A}_r^l(m) \cos\left[\hat{\theta}_r^l(m)\right]$$

$$d(n) = \sum_{l=0}^{L-1} d^l(n - lH)$$

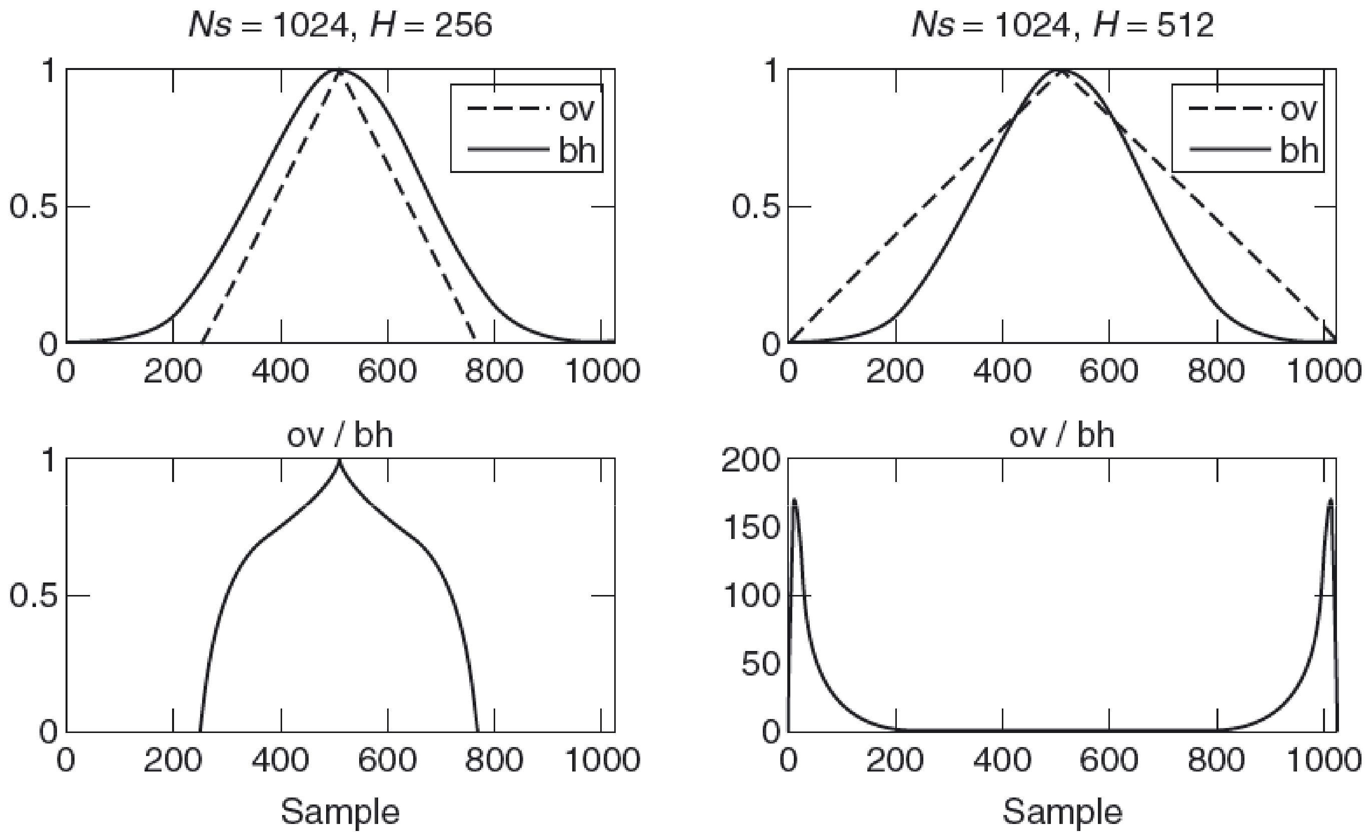


**Figure 10.12** Overlapping strategy based on dividing by the synthesis window and multiplying by an overlapping window. Undesired artifacts can be introduced at synthesis if the overlap is not significantly greater than 50%.

$$w_{BH92}(n) = 0.35875 + 0.48829 \cos\left(\frac{2\pi n}{N}\right) + 0.14128 \cos\left(\frac{4\pi n}{N}\right) + \\ +0.01168 \cos\left(\frac{6\pi n}{N}\right),$$

# 10.3.4 Spectral harmonics

$$d(\tau) = \sum_{j=t+1}^{t+W} \left(x(j) - x(j+\tau)\right)^2$$

$$d'(\tau) = \begin{cases} 1 & if\ \tau = 0 \\ \frac{\tau \cdot d(\tau)}{\sum_{j=1}^{\tau} d(j)} & otherwise \end{cases}$$

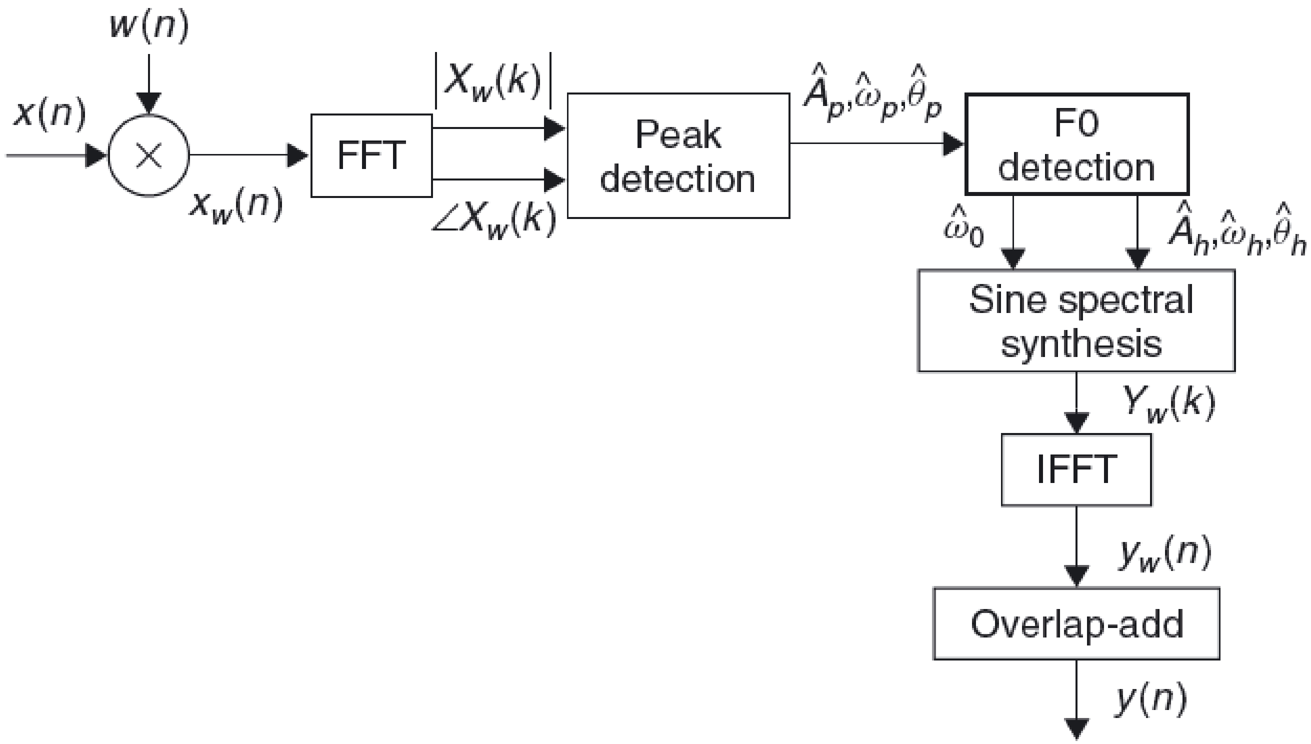


**Figure 10.13** Block diagram of an analysis/synthesis system based on the harmonic sinusoidal model.

# 10.3.5 Spectral harmonics plus residual

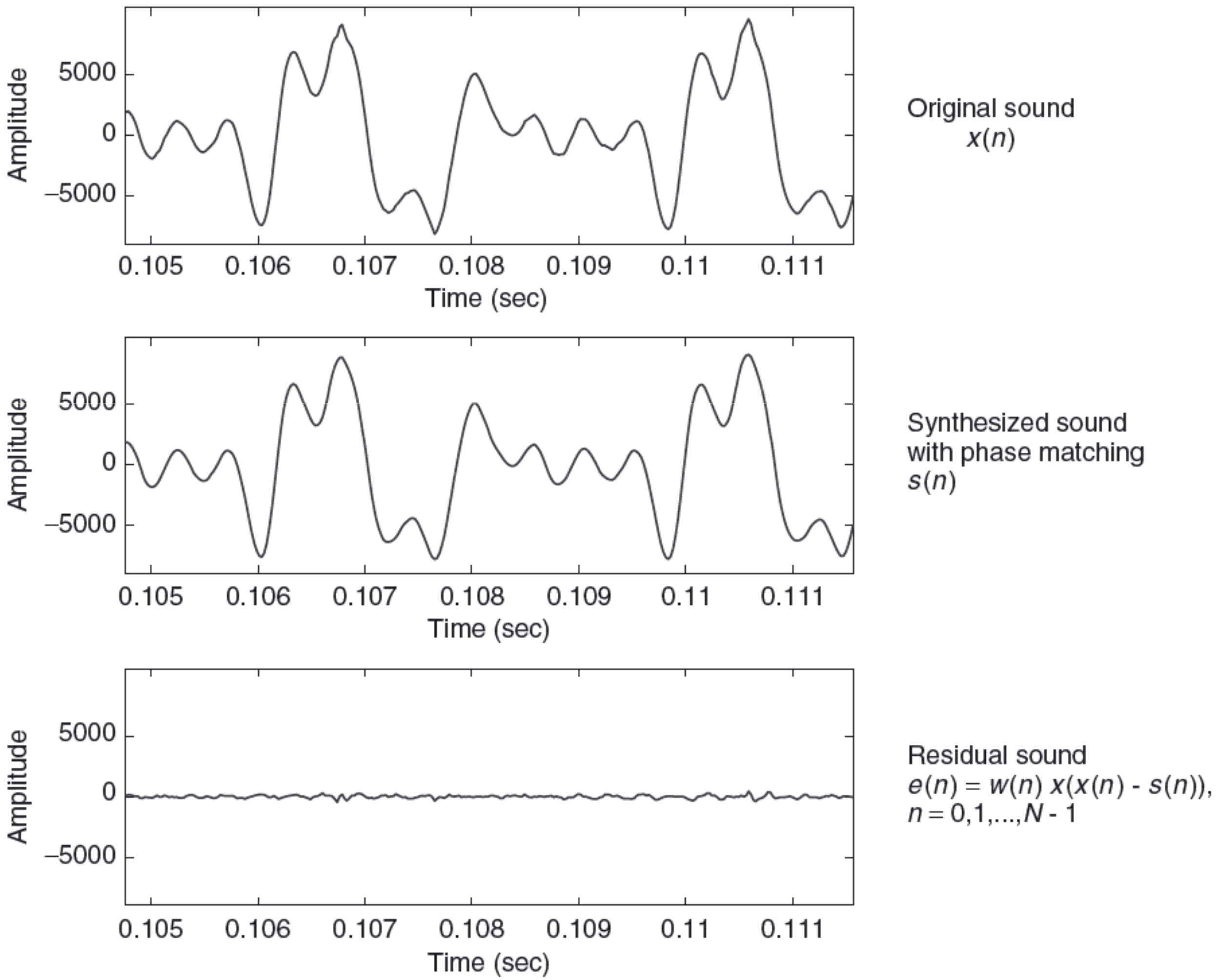


**Figure 10.14** Time-domain subtraction.

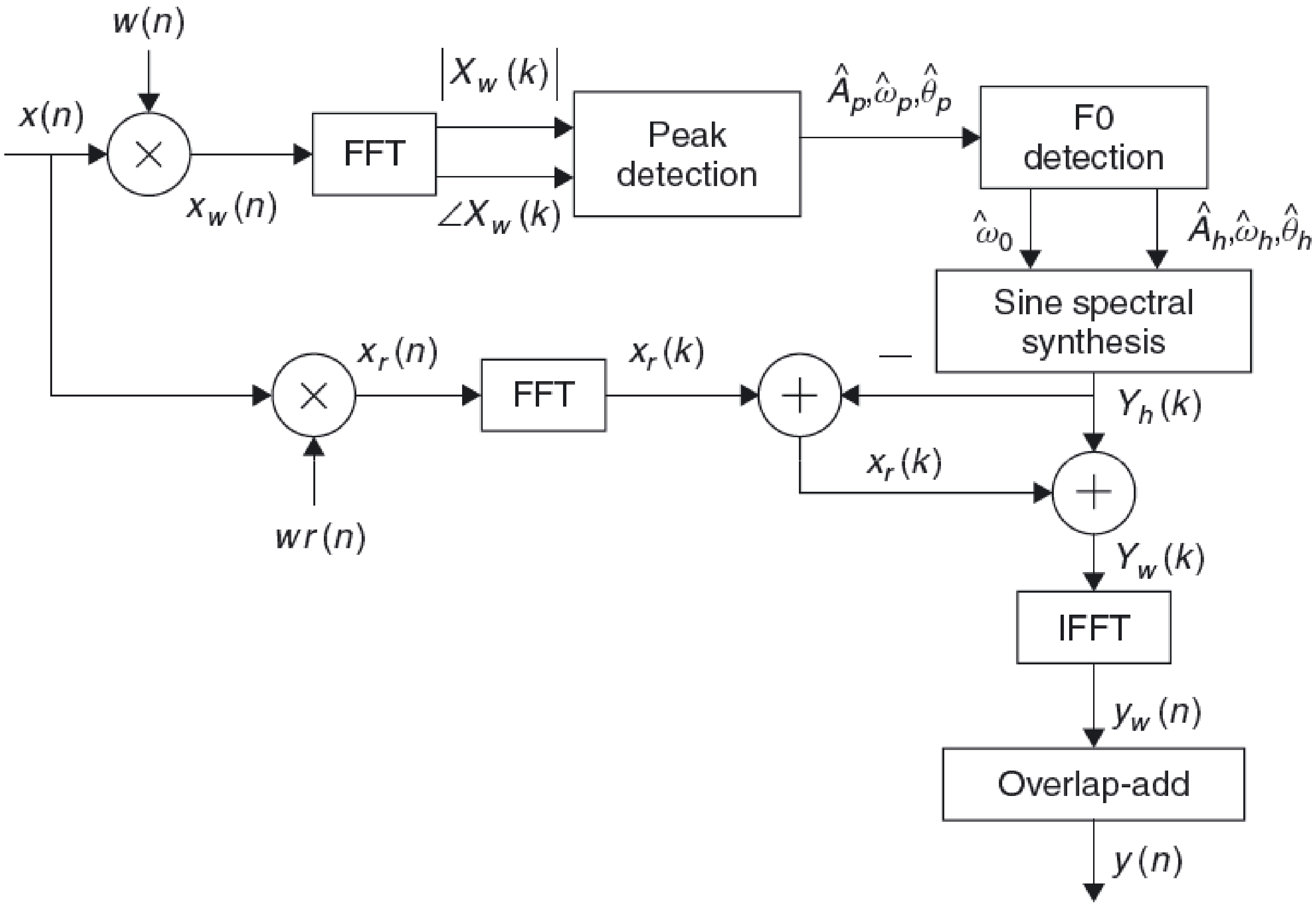


**Figure 10.15** Block diagram of the harmonic plus residual analysis/synthesis system.

# 10.3.6 Spectral harmonics plus stochastic residual

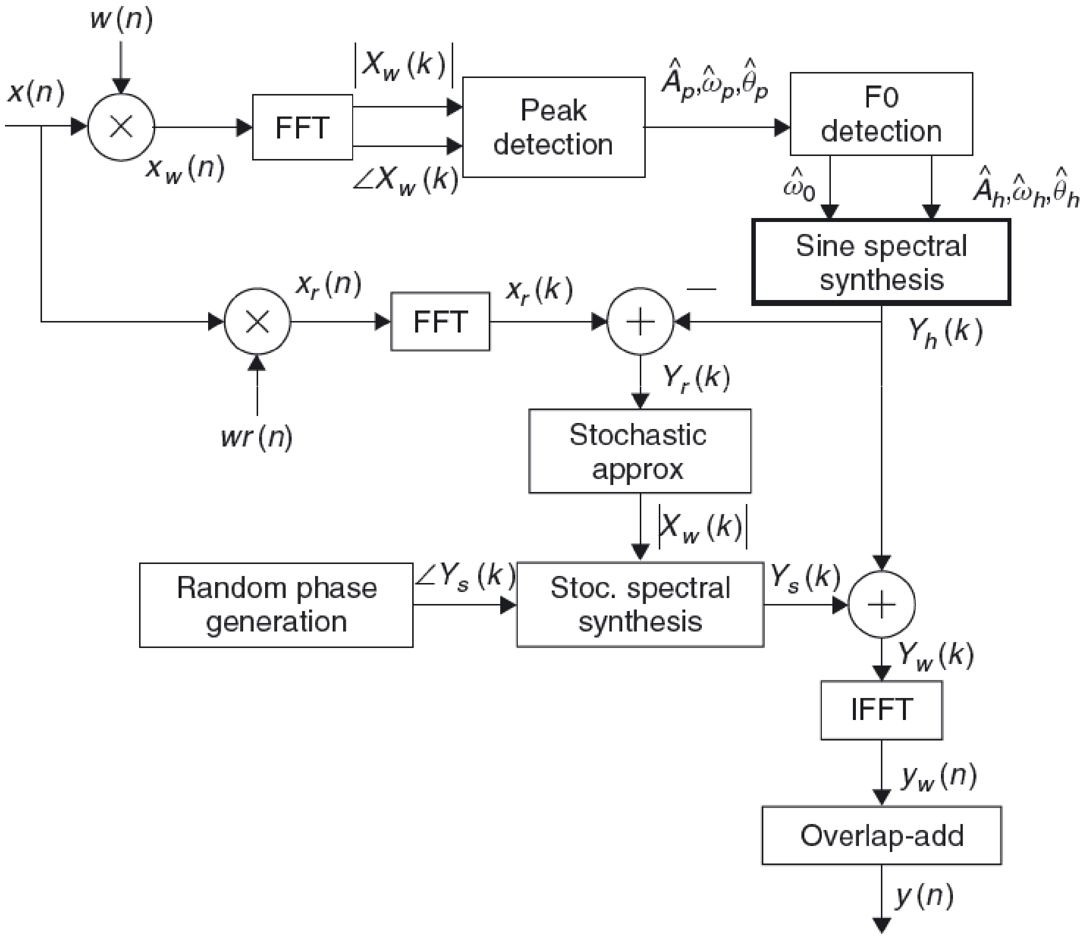


**Figure 10.16** Block diagram of an analysis/synthesis system based on a harmonic plus stochastic model.

- **Residual analysis**

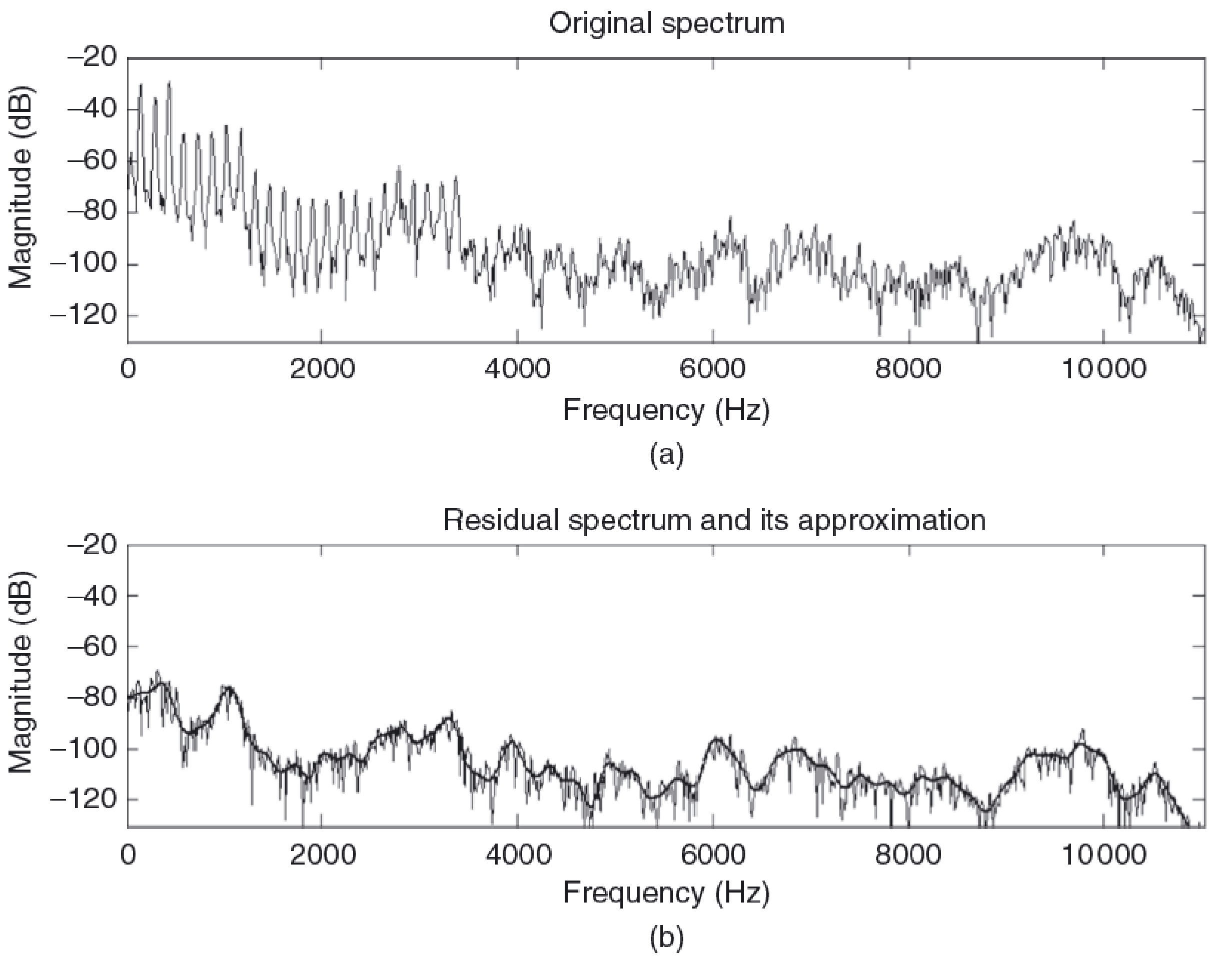


**Figure 10.17** (a) Original spectrum. (b) Residual spectrum and approximation.

• **Residual synthesis**

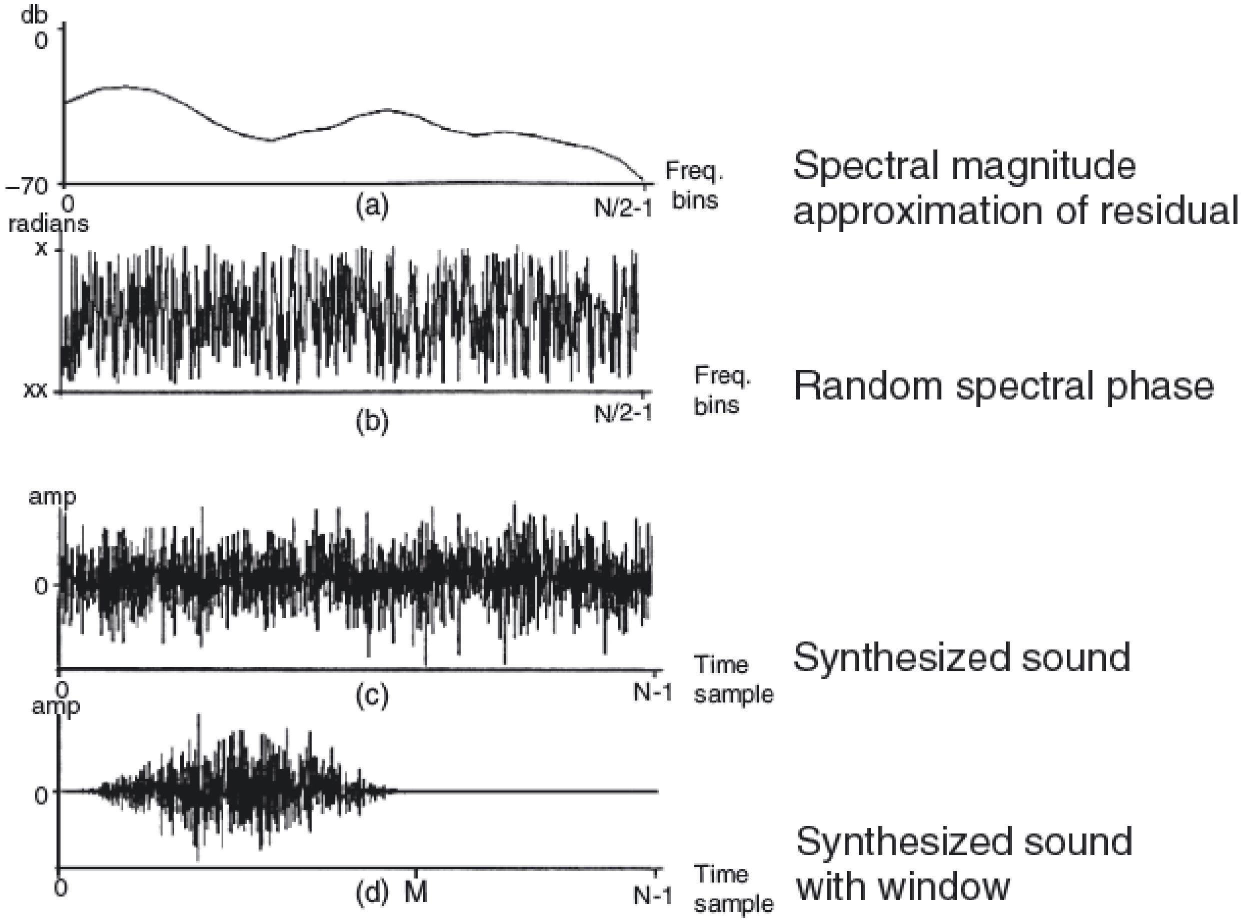


**Figure 10.18** Stochastic synthesis.

# 10.4 Effects

## 10.4.1 Sinusoidal plus residual

- **Filtering**

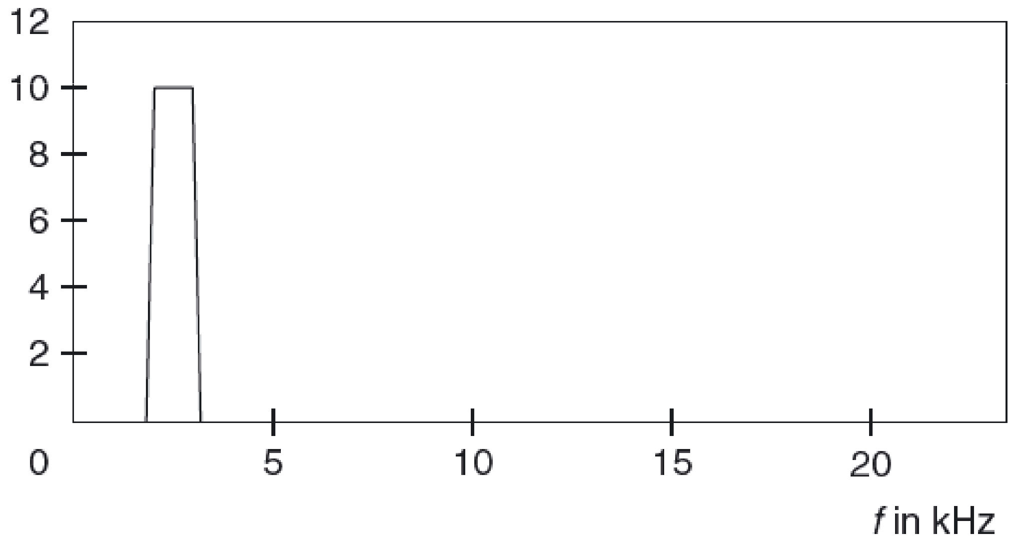


**Figure 10.19** Bandpass filter with arbitrary resolution.

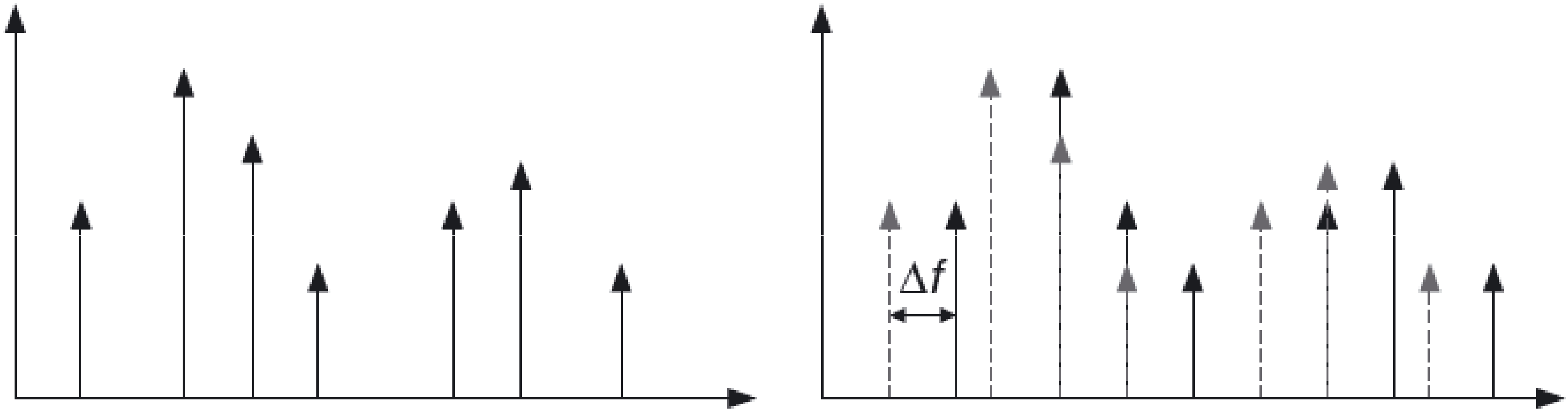


**Figure 10.20** Frequency shift of the partials.

- **Frequency shifting**
- **Frequency stretching**

$$f_i = f_i \cdot f_{stretch}^{(i-1)}$$

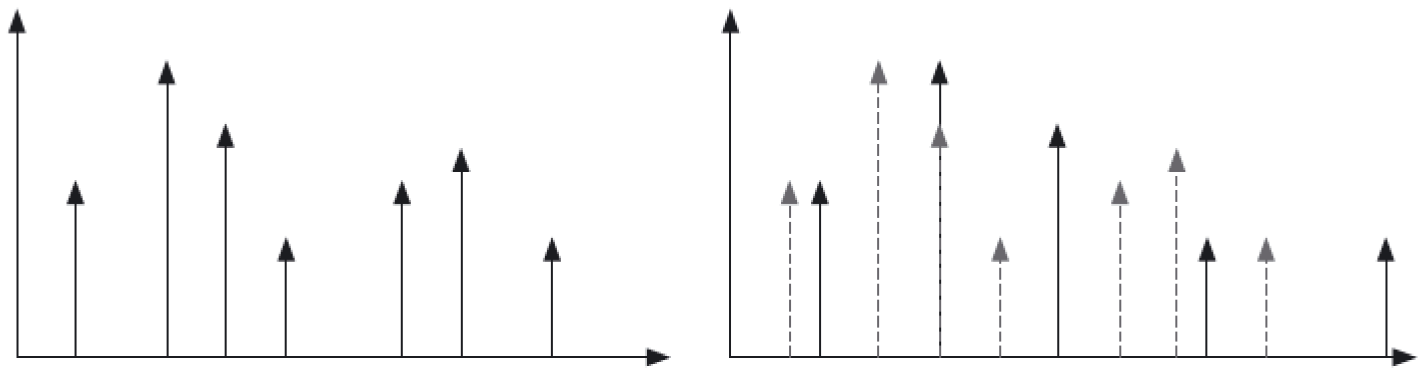

**Figure 10.21** Frequency stretching.

- **Frequency scaling**
- **Time scaling**

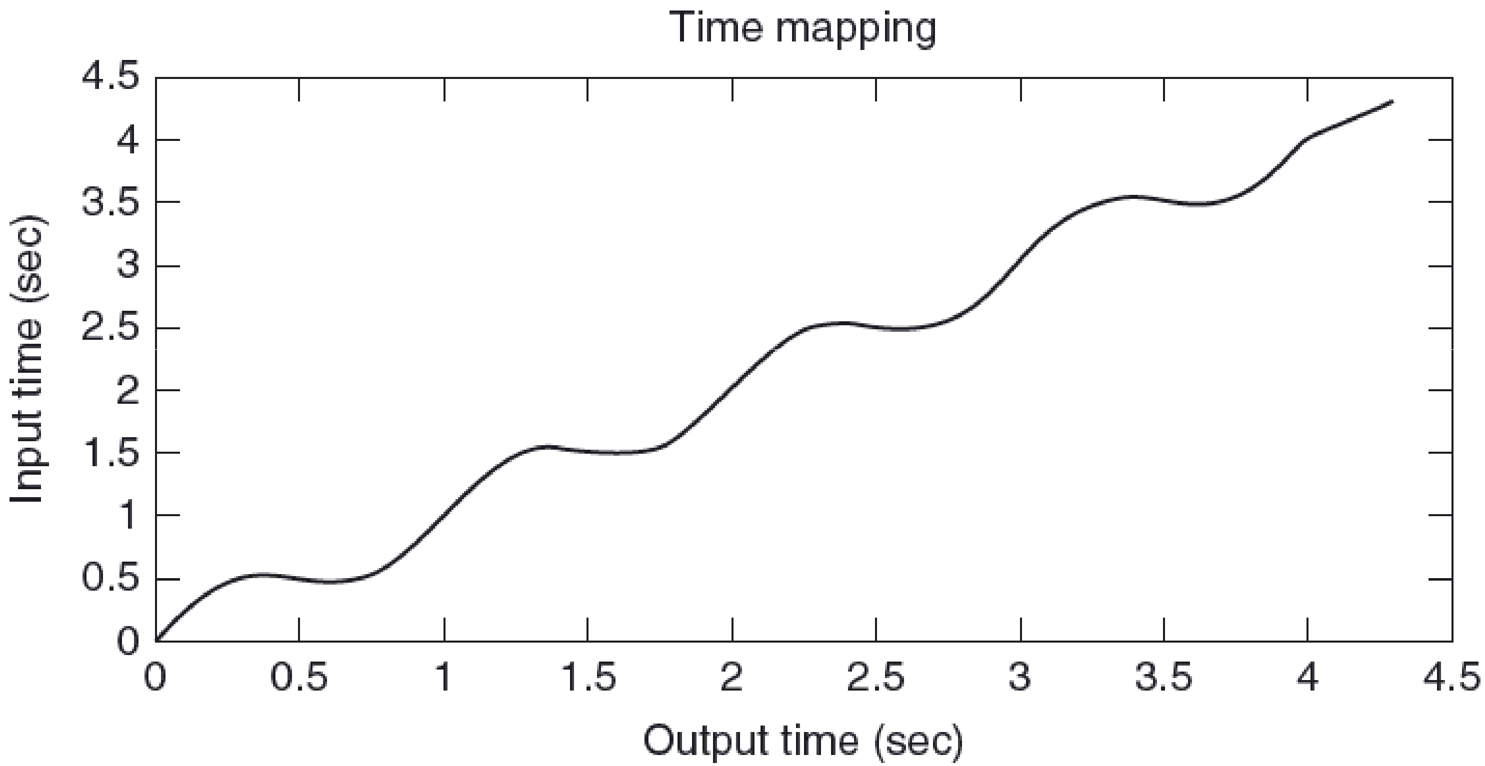


**Figure 10.22** Complex time mapping.

# 10.4.2 Harmonic plus residual

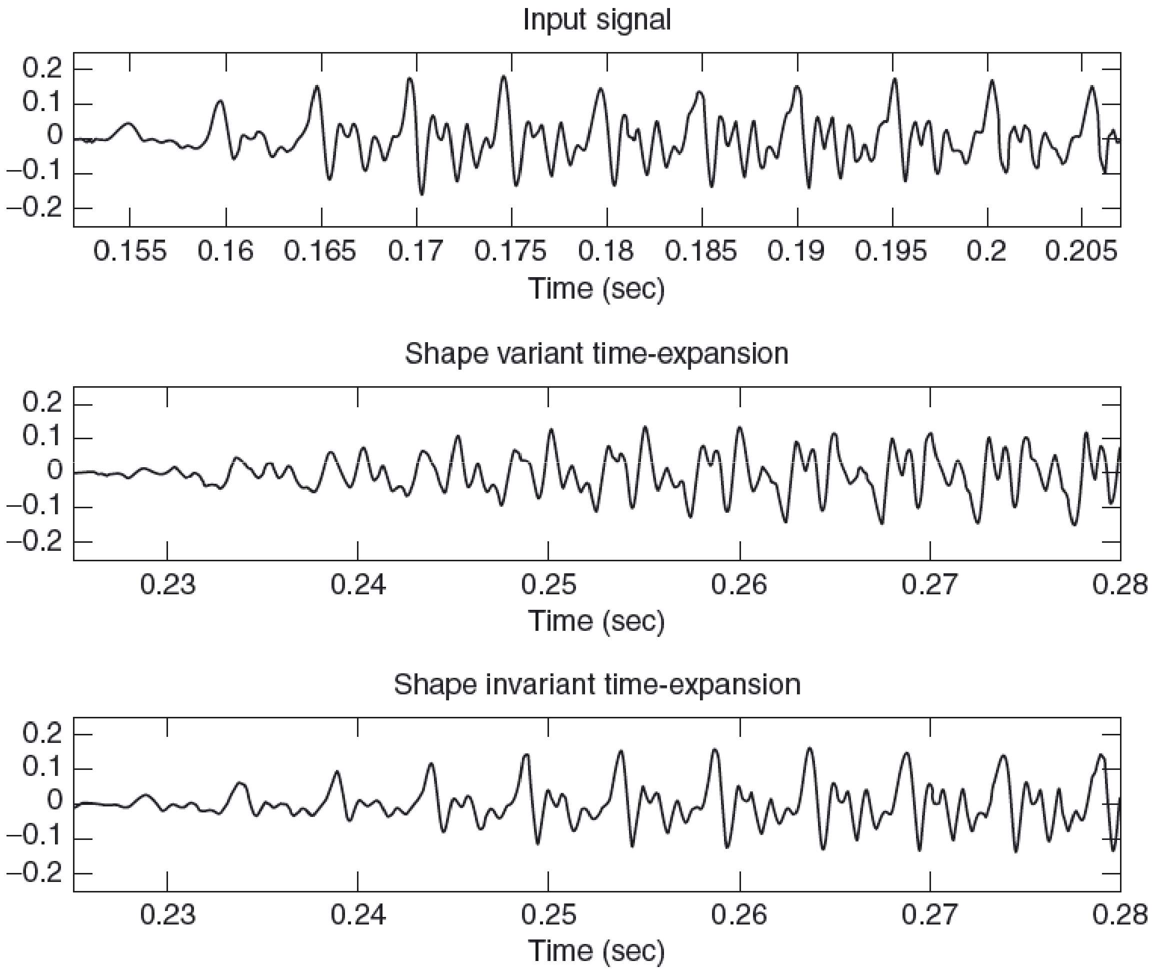


**Figure 10.23** Shape-variant and -invariant transformations. In this example the input signal is time-expanded by a factor of 1.5.

- **Harmonic filtering**
- **Pitch discretization**

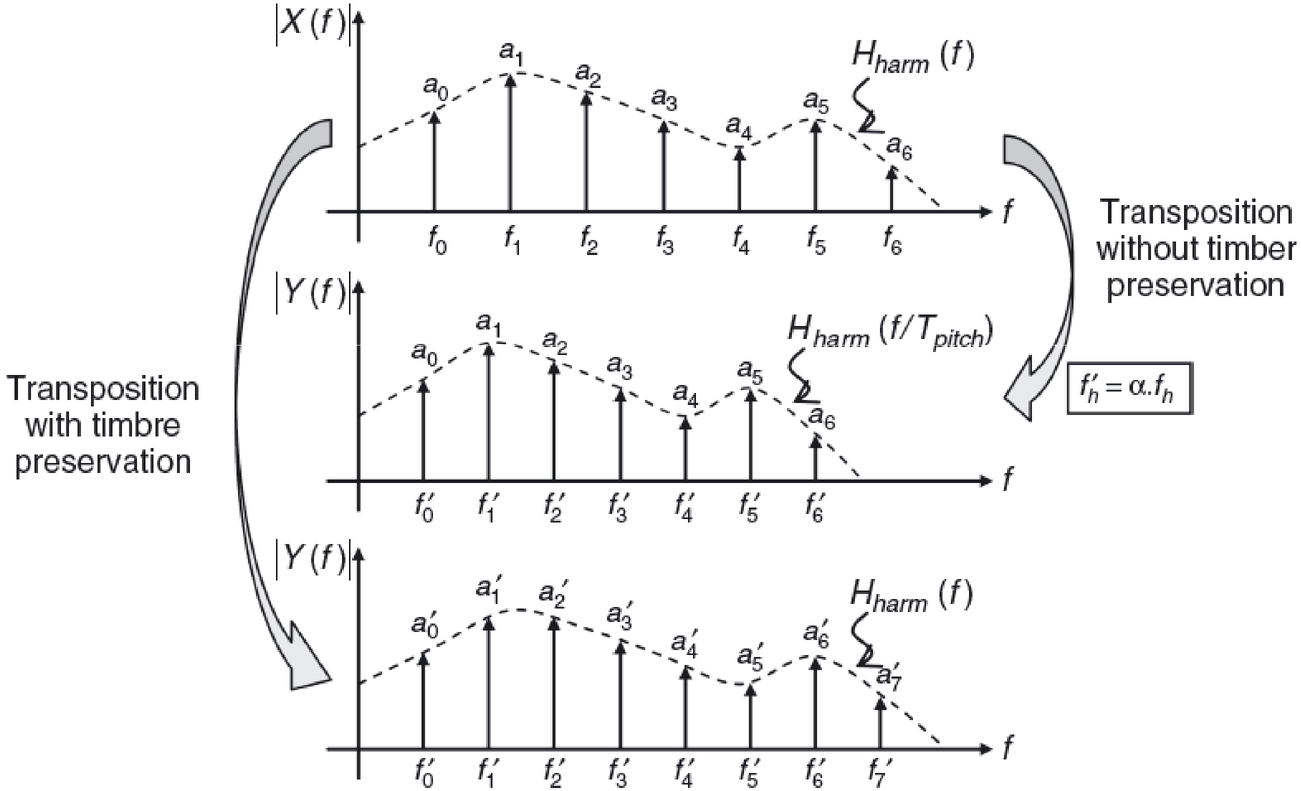


**Figure 10.24** Pitch transposition. The signal whose transform is represented in the top view is transposed to a lower pitch. The middle view shows the result when only harmonic frequencies are modified, whereas in the bottom view representation both harmonic frequencies and amplitudes have been modified so that the timbre is preserved.

- **Pitch transposition with timbre preservation**
- **Vibrato and tremolo**
- **Timbre scaling**
- **Roughness**

# 10.4.3 Combined effects

- **Gender change**
- **Harmonizer**
- **Choir**
- **Morphing**

# 10.5 Conclusions